Name _____ Date _____

Pre-Calculus H – Donovan

Summer Assignment 2023 – 2024

Order of Topics:

A. Lines
B. Factoring / Solving Equations by Factoring
C. Solving Equations Using the Quadratic Formula *(Calculator Allowed)*
D. Solving Mixed Equations
E. Logarithms & Exponential Equations
F. Domain
G. Exponents
H. Inverses
I. Function Notation
J. Function Operations & Compositions
K. Radicals
L. Extra Topics

A calculator should only be used on topics (above) OR individual problems in the packet that state "Calculator Allowed".

You are responsible for knowing and understanding all of the topics in this packet (they are all from Algebra I, Geometry, & Algebra II). **This packet itself will not be graded.** In September, we will have two days, as a class, to go over any topics you are having trouble with (so 96 minutes of class time review) and then you will have a two-day test on this content that will count as your first test grade in marking period 1. I will send out the answer key with all the work included by the beginning of August so you can review it yourself and come in, ready with questions.

Round or truncate every answer to 3 decimal places, as that is what the AP Calculus test requires.

	Exact Answer	**Approximate Answer**
Find the circumference of a circle with a radius of 4 cm.	$C = 8\pi$ cm	$C = 25.133$ cm
Simplify $\sqrt{40}$	$2\sqrt{10}$	6.325

A. Lines

Equations of Lines:

- **Slope-Intercept Form: $y = mx + b$, where m = slope and b = y-intercept**
- **Point-Slope Form: $y - y_1 = m(x - x_1)$, where m = slope and (x_1, y_1) is a point on the line**
- **Standard Form: $Ax + By = C$, where A is a positive integer, B & C are integers, and A, B, and C are relatively prime (have a greatest common factor of 1)**

- **Horizontal lines have an equation of $y = \#$ and have a slope of 0**
- **Vertical lines have an equation of $x = \#$ and have an undefined slope**

1. Write the equation of the line parallel to $2x - 6y = -1$ and containing the x-intercept of $4x - 3y = 12$. Write your answer in slope-intercept form.

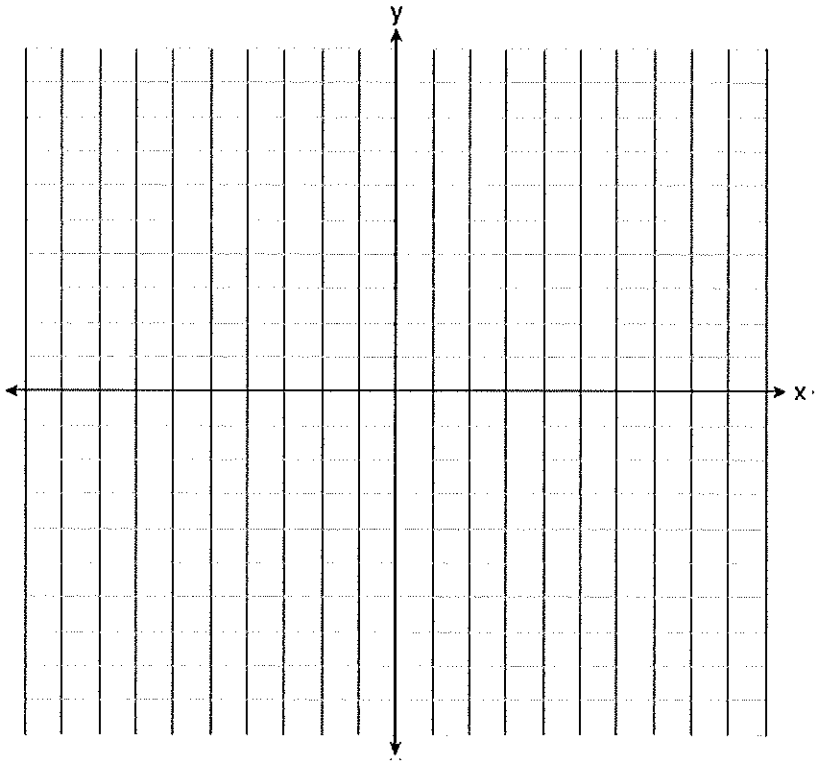
2. Write the equation of the line through the point with coordinates $(-4, 6)$ and perpendicular to $3x - 2y = 8$. Write your answer in standard form.

3. Find the value of "a" if a line containing the point $(a, -2a)$ has a y-intercept of 6 and a slope of $-\frac{2}{3}$.

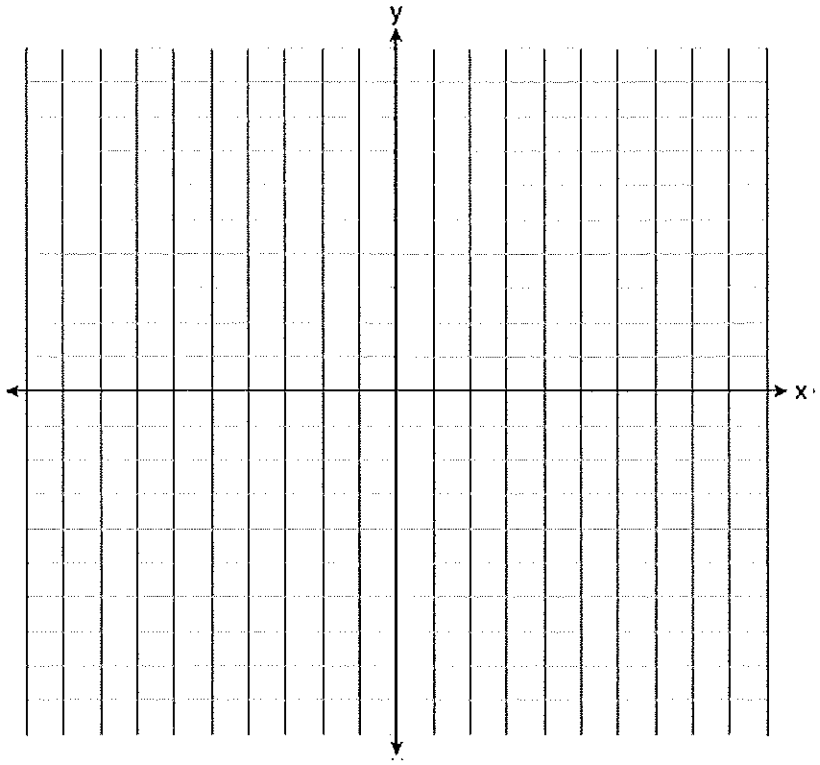
4. Write the equation of the perpendicular bisector of the segment joining the points with coordinates of $(-3, 4)$ and $(5, -2)$. Write your answer in point-slope form.

5. Find the x- and y-intercepts of the equation $5x + 3y = -15$.

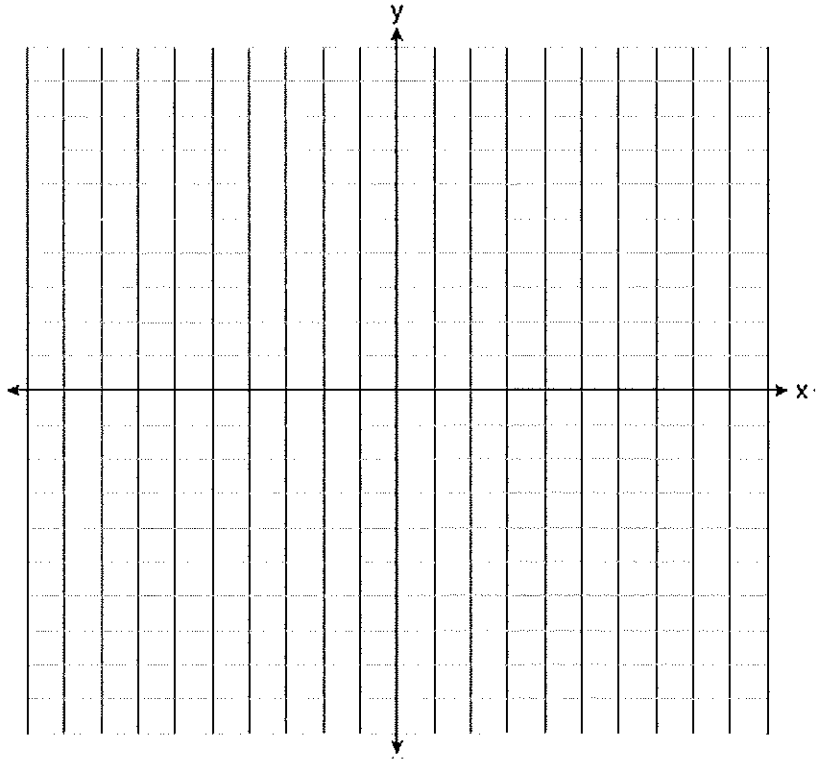
6. Graph the line $y = -2x$



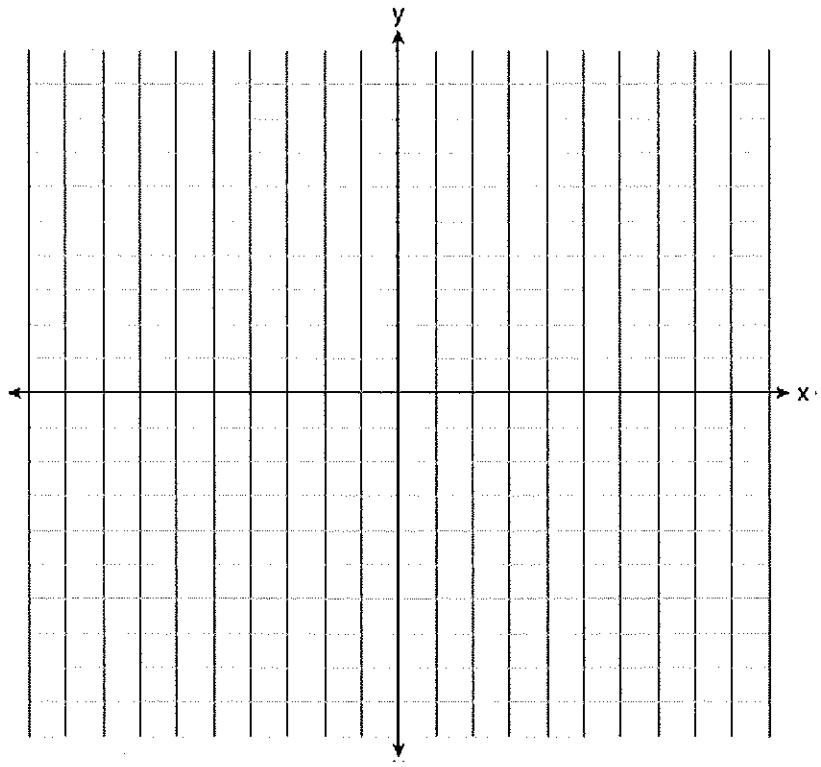
7. Graph the line $y - 4 = -\frac{1}{2}(x - 3)$



8. Graph the line $x = 0$



9. Graph the line $2x - 5y = 15$



B. Factoring / Solving Equations by Factoring

When factoring, always look for a greatest common factor first.

- **If the expression has 2 terms, check if it can be factored using:**
  - **Difference of Two Squares:** $(a^2 - b^2) = (a + b)(a - b)$
  - **Sum of Two Cubes:** $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
  - **Difference of Two Cubes: :** $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
- **If the expression has 3 terms** $(ax^2 + bx + c)$**, check if it can be factored using:**
  - **Multiply to** ac**, add to** b**, then factor by grouping**
  - **Perfect square trinomial:** $a^2 + 2ab + b^2 = (a + b)^2$ **or** $a^2 - 2ab + b^2 = (a - b)^2$
- **If the expression has 4 terms, check if it can be factored using:**
  - **Grouping**

Factor each of the following completely.

10. $6x^2 - 11x + 3$

11. $16a^4 - 81y^8$

12. $6x^2 - 13x - 5$

13. $6ax + 6xc + ba + bc$

14. $x^3 + 2x^2 - x - 2$

15. $27x^3 + 1$

16. $x^6 + y^{12}$

17. $m^6 - 8$

18. $2x^2 + xy - 6y^2$

19. $9x^2 - 16y^2$

20. $225x^4 - 64y^8$

21. $x^6 - 1$

22. $y^6 + 216$

23. $x^3 + y^3 - x^2y - xy^2$

24. $x^{12} - y^{24}$

Solve each equation by factoring.

25. $(3n - 2)(4n + 1) = 0$

26. $m^2 - 3m = 0$

27. $3k^2 + 72 = 33k$

28. $n^2 = -18 - 9n$

29. $b^2 + b = 2$

30. $10n^2 - 35 = 65n$

31. $3x^2 - 8x = 16$

32. $4x^2 - 11x - 3 = 0$

C. Solving Equations Using the Quadratic Formula *(Calculator Allowed)*

For solving quadratic equations, remember:

- **The quadratic formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- **The discriminant,** $b^2 - 4ac$**, will tell you how many solutions the quadratic has:**

$$b^2 - 4ac = \begin{cases} > 0, \text{2 real solutions} \\ = 0, \text{1 real solution} \\ < 0, \text{0 real solutions (2 imaginary solutions)} \end{cases}$$

Solve each equation using the quadratic formula.

33. $2x^2 - x - 13 = 2$

34. $8x^2 - 4x = 18$

35. $10x^2 + 9 = x$

36. $x^2 = 9x - 20$

37. $9x^2 - 11 = 6x$

38. $2x^2 - 14x + 40 = 3x^2 - 16x + 32$

D. <u>Solving Mixed Equations</u>

Solve each equation.

39. $\frac{x}{2x+7} - \frac{1}{x-2} = 0$

40. $|3x + 2| = 4$

41. $\sqrt{3 - 2x} = \sqrt{1 - 3x}$

42. $\sqrt{2v - 7} = v - 3$

43. $2 = \sqrt{3b - 2} - \sqrt{10 - b}$

E. Logarithms & Exponential Equations

- If $y = b^x$, then $x = \log_b y$.
- It the base of a log is not specified, it is defined to be 10. When we are asked to solve $\log 100$, we are solving the equation: $10^x = 100$ and $x = 2$.
- $\ln e = 1$
- $\ln 1 = 0$
- $e^{\ln x} = x$
- The function $y = \log x$ has domain $(0, \infty)$ and range $(-\infty, \infty)$.
- Finding $\ln 5$ is the same as solving the equation $e^x = 5$.
- Rules for simplifying problems involving logs and natural logs (ln) are below. These rules work with logs of any base including natural logs.
  - $\log a + \log b = \log(a \cdot b)$
  - $\log a - \log b = \log\left(\frac{a}{b}\right)$
  - $\log a^b = b \log a$
  - $\log_b b = 1$
  - $\log_b b^n = n$
  - $b^{\log_b n} = n$

Evaluate each expression.

44. $\log_5 1$

45. $\log_{\frac{1}{25}} 5$

46. $\log_4 128$

47. $\log_4 \frac{1}{16}$

48. $\log_9 27$

49. $\log_{27} \frac{1}{3}$

Solve each equation. Check for extraneous solutions.

50. $\log_9 x = \frac{3}{2}$

51. $\log_{10} x^2 = -4$

52. $\log_4(2x) = -\frac{1}{2}$

53. $\log_4 x = -\frac{3}{2}$

54. $\log_6(2x - 1) = 3$

55. $\log_8(x - 5) = \frac{2}{3}$

56. $\log_6(2x - 3) = \log_6 12 - \log_6 3$

57. $\log(x + 2) - \log x = 2 \log 4$

58. $3\log_2 x - 2\log_2(5x) = 2$

59. $\log x - \log(x+6) = \frac{1}{2}\log 9$

60. $\log_2(x+7) + \log_2 x = 3$

61. $e^{3-5x} = 16$

62. $e^{2x+1} = 200$

63. $10^{1-x} = 4$

F. Domain

The domain of a function is the set of allowable x-values. The domain of a function f is $(-\infty, \infty)$ except for values of x which create a zero in the denominator, an even root of a negative number, or a logarithm of a non-positive number.

Find the domain of each function.

64. $g(x) = 4x - 3$

65. $g(x) = \frac{4}{x^2-4}$

66. $g(x) = \frac{x+1}{x^2+4x}$

67. $h(x) = \frac{x+2}{\sqrt{9-x^2}}$

68. $f(x) = \frac{x+5}{x+4}$

69. $j(x) = \frac{x-2}{x^2-16x+60}$

70. $r(x) = \sqrt{x-2}$

71. $f(x) = \frac{5}{|x+4|}$

72. $\log_4(x-7)$

G. Exponents

- **Negative powers do not make expressions negative; they create fractions.**

$$x^{-n} = \frac{1}{x^n}, x \neq 0$$

- **Fractional exponents create roots.**

$$x^{1/2} = \sqrt{x} \text{ and } x^{a/b} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$$

- **When we multiply terms with the same base, we add exponents:** $(x^a)(x^b) = x^{a+b}$
- **When we divide terms with the same base, we subtract exponents:** $\frac{x^a}{x^b} = x^{a-b}$ **or** $\frac{x^a}{x^b} = \frac{1}{x^{b-a}} x \neq 0$
- **When we raise powers, we multiply exponents:** $(x^a)^b = x^{ab}$

Simplify using exponent rules. Write your answer using positive exponents.

73. $\frac{x^{-2}y}{x^4y^{-1}}$

74. $\frac{12m^8y^6}{-9my^4}$

75. $(4a^3c^2)^3(-3ac^4)^2$

76. $\left(\frac{5a^7}{2b^5c}\right)$

77. $\left(\frac{7m^{-1}n^3}{m^{-1}n^2}\right)^{-1}$

78. $\frac{\left((3x^{-2}y^3)(5xy^{-8})\right)}{(x^3)^4y^{-2}}$

79. $\frac{4^{-5}4^{6}}{4^{2}}$

H. Inverses

- **To find the inverse of a function, switch x and y and solve for the new y.**
- **To prove that one function is an inverse of another function, you need to show that $f(g(x)) = g(f(x)) = x$.**

Find the inverse of each function.

80. $f(x) = x^3$

81. $h(x) = \frac{1}{x}$

82. $w(x) = 2x + 1$

83. $g(x) = x^2 + 1, x \geq 0$

84. $r(x) = \sqrt[5]{2x+1}$

I. Function Notation

- **A function is a set of points (x, y) such that for every x, there is one and only one y.**
- **To evaluate a function for a given value, plug the value into the function for x.**

Find the value of each.

Given: $f(x) = 3x - 7, g(x) = x^2 + 3, h(x) = \begin{cases} 3x \text{ if } x < 0 \\ x + 1 \text{ if } 0 \leq x \leq 2 \\ (x-2)^2 \text{ if } x > 2 \end{cases}$

85. $f(-1)$

86. $f(x+3)$

87. $h(-5)$

88. $f\big(f(x)\big)$

89. $g(x+2)-g(x)$

90. $f\big(g(2)\big)$

91. $h(0)$

92. $g\big(f(-3)\big)$

93. $f\big(h(4)\big)$

J. Function Operations & Compositions

- **The four basic function operations:**
  - **Addition:** $(f+g)(x) = f(x) + g(x)$
  - **Subtraction:** $(f-g)(x) = f(x) - g(x)$
  - **Multiplication:** $(fg)(x) = f(x) \cdot g(x)$
  - **Division:** $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

- $(f \circ g)(x) = f\big(g(x)\big) = f[g(x)]$, **read "$f$ of g of x", means to plug the inside function in for x in the outside function.**

Find $(f+g)(x), (f-g)(x), (f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each of the following function pairs. Find the domain of each.

94. $f(x) = \frac{1}{x}, g(x) = 7 - x$

a. Find $(fg)(2)$

95. $f(x) = \frac{1}{2-3x}, g(x) = \frac{2}{3x-2}$

a. Find $(f + g)(-3)$

96. $f(x) = \frac{3x+5}{2}, g(x) = \frac{2x-5}{3}$

a. Find $\left(\frac{f}{g}\right)(-1)$

Find $[f \circ g](x)$ and $[g \circ f](x)$.

97. $f(x) = x^2, g(x) = \frac{1}{x^3}$

98. $f(x) = \frac{x}{x-2}, g(x) = \frac{3}{x}$

99. $f(x) = \frac{x-1}{x-2}, g(x) = \frac{x-3}{x-4}$

100. $f(x) = x^2 - 16, g(x) = \sqrt{x}$

K. Radicals

Simplify the following. Rationalize all denominators.

101. $\sqrt[3]{24}$

102. $\sqrt[3]{-40x^6y^7}$

103. $\frac{5}{2\sqrt{3}}$

104. $\sqrt{75x^3} \cdot \sqrt{5x^3}$

105. $2\sqrt{48} - 3\sqrt{27}$

106. $\frac{5-\sqrt{2}}{3+\sqrt{2}}$

107. $(\sqrt{6}-18)(2\sqrt{3}-9)$

L. Extra Topics

- **Distance Formula:** $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
- **Midpoint Formula:** $x=\frac{x_1+x_2}{2}; y=\frac{y_1+y_2}{2}$
- **In both of these formulas, (x_1, y_1) represents the first ordered pair and (x_2, y_2) represents the second ordered pair**

108. Find the distance and midpoint between the points $(-3, 5)$ and $(4, 7)$. *(Calculator Allowed)*

109. Simplify

$$\frac{(4+6i)}{3i}$$

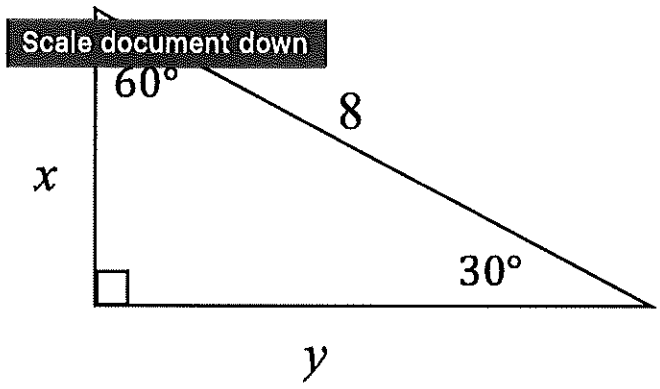
110. Simplify.

$$(4-i)(4+i)$$

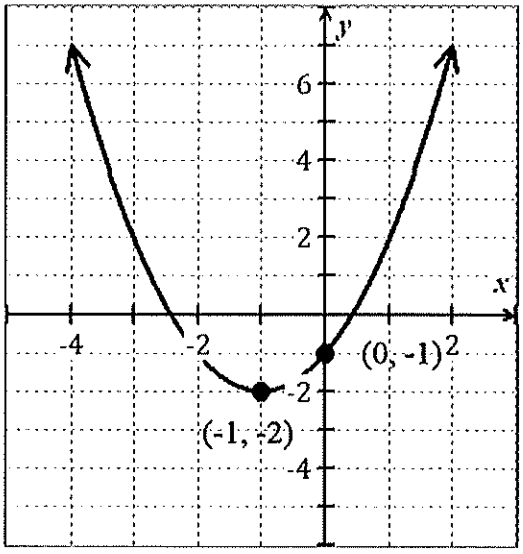
111. Solve for m.

$$g - 4cm = -3m$$

112. Find the exact value of x and y.



113. Name the equation of the function graphed.



114. Simplify.

$$\frac{x^2 - 13x + 40}{x^2 - 25}$$